Eur. Phys. J. A **15**, 421–427 (2002)
DOI 10.1140/epja/i2002-10080-2

The European Physical Journal A

Short Note

Momentum distribution of relativistic nuclei with Hartree-Fock mesonic correlations

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Received: 7 June 2002 / Revised version: 9 September 2002 /
Published online: 10 December 2002 –
Communicated by G. Orlandini

Abstract. The impact of Hartree-Fock correlations on the nuclear momentum distribution is studied in a fully relativistic one-boson-exchange model. Hartree-Fock equations are exactly solved to first order in the coupling constants. The renormalization of the Dirac spinors in the medium is shown to affect the momentum distribution, as opposed to what happens in the non-relativistic case. The unitarity of the model is shown to be preserved by the present renormalization procedure.

PACS. 21.60.Jz Hartree-Fock and random-phase approximations – 21.65.+f Nuclear matter – 24.10.Jv Relativistic models – 11.10.Gh Renormalization

It is well known that in a non-relativistic framework the momentum distribution of nuclear matter is not affected by the Hartree-Fock (HF) field. This arises because, due to general invariance principles [1], the non-relativistic self-energy cannot depend on spin in an infinite system: hence the single-nucleon wave functions are not modified and only the energy-momentum relation is affected by the medium. Of course, correlations in the nuclear wave function beyond the mean-field approximation are very important already at the non-relativistic level [2–4]. Due to such correlations, the momentum distribution is reduced for momenta below $k_{\rm F}$ and the states above $k_{\rm F}$ acquire small but finite occupation probabilities.

Although the momentum distribution is not an observable, it is also true that over the years electron scattering reactions have frequently been expressed in terms of momentum densities. In recent work [5,6] we have evaluated the impact of mesonic correlations and meson exchange currents (MEC) on the electroweak response functions within a fully relativistic, gauge-invariant model. We have shown that the consistency of the theory necessarily implies the inclusion in the calculation of Hartree-Fock self-energy insertions. In order to deal properly with the divergencies associated with these diagrams, not only the energy but also the nucleon wave functions must be renormalized by the medium. As a consequence, in a relativistic HF framework, the momentum distribution is also modified for $k < k_{\rm F}$, since now the Dirac four-spinors describing the nucleons display new features and the self-energy becomes spin dependent. The aim of this paper is to quantify this genuine relativistic effect in a one-boson-exchange model for the NN interaction, while the corresponding observable consequences on the response functions were analyzed in depth in [5,6].

An unambiguous treatment of relativistic Hartree-Fock does not exist in the literature, since the presence of the Dirac sea requires (at least) the specification of a prescription to take it into account [7]. The approach we use is equivalent to that used in [7,8], where the nucleon proper self-energy is calculated in terms of positive-energy spinors only. This approximation is valid in the first iteration of a fully self-consistent calculation to which we confine ourselves in this work. This procedure was shown in [7] to reproduce the non-relativistic HF equations in the limit $M \to \infty$, and it reduces the relativistic Hartree approximation to the mean-field theory.

The proton momentum distribution of nuclear matter in the independent-particle approximation is

$$n(\mathbf{p}) = \sum_{\mathbf{k},s} \psi^\dagger_{\mathbf{k},s}(\mathbf{p})\psi_{\mathbf{k},s}(\mathbf{p})\theta(k_{\rm F}-k) \ , \qquad (1)$$

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where $k_{\rm F}$ is the Fermi momentum. Since we are focusing on symmetric nuclear matter the neutron and proton momentum distributions are equal. For a free relativistic Fermi gas, in momentum space the wave function describing a nucleon with momentum $\mathbf{k}$ and spin s is given by

$$\psi_{\mathbf{k},s}(\mathbf{p}) = \int_V \mathrm{d}\mathbf{r}\, \psi_{\mathbf{k},s}(\mathbf{r}) e^{-i\mathbf{p}\cdot\mathbf{r}} = \sqrt{\frac{m}{VE_{\mathbf{k}}}}\, u_s(\mathbf{k},m) \int_V \mathrm{d}\mathbf{r} e^{i(\mathbf{k}-\mathbf{p})\cdot\mathbf{r}} = \sqrt{\frac{Vm}{E_{\mathbf{k}}}}\, u_s(\mathbf{p},m)\delta_{\mathbf{k},\mathbf{p}} , \tag{2}$$

where V is the volume enclosing the system, $u_s(\mathbf{k},m)$ is the free Dirac spinor ($\not{K} u_s = m u_s$) and $E_{\mathbf{k}} = \sqrt{\mathbf{k}^2+m^2}$ is the free energy of the nucleon. We use the Bjorken and Drell [9] conventions for the spinor normalization, $\bar{u}u = 1$. Therefore, the wave function in coordinate space is normalized to one: $\int_V \mathrm{d}\mathbf{r}\psi^\dagger_{\mathbf{k},s}(\mathbf{r})\psi_{\mathbf{k},s}(\mathbf{r}) = 1$.

The wave function of eq. (2), inserted in eq. (1), yields the well-known result

$$n(\mathbf{p}) = V\sum_{\mathbf{k},s} \frac{m}{E_{\mathbf{k}}} u^\dagger_s(\mathbf{p},m)u_s(\mathbf{p},m)\theta(k_{\rm F}-p)\delta_{\mathbf{p},\mathbf{k}} = 2V\theta(k_{\rm F}-p) . \tag{3}$$

In an interacting system, in the relativistic HF approximation, the above distribution is modified, since the single-particle wave functions are renormalized by the interaction with the other nucleons in the medium. In this case, the Dirac equation in the nuclear medium is given by

$$[\not{P} - m - \Sigma(P)]\widetilde{\phi}_s(\mathbf{p}) = 0 , \tag{4}$$

where $\widetilde{\phi}_s(\mathbf{p})$ is the renormalized spinor and $\Sigma(P)$ is the self-energy of a nucleon in nuclear matter. According to general symmetry properties $\Sigma(P)$ can be written in the form [8,10]

$$\Sigma(P) = mA(P) + B(P)\gamma_0 p_0 - C(P)\boldsymbol{\gamma}\cdot\mathbf{p} . \tag{5}$$

Using the above decomposition the Dirac equation (4) can be recast as

$$[1-C(P)]\left[\gamma_0 f_0(P) - \boldsymbol{\gamma}\cdot\mathbf{p} - \widetilde{m}(P)\right]\widetilde{\phi}_s(\mathbf{p}) = 0 , \tag{6}$$

where the functions

$$f_0(P) = \frac{1-B(P)}{1-C(P)}\, p_0 , \tag{7}$$

$$\widetilde{m}(P) = \frac{1+A(P)}{1-C(P)}\, m \tag{8}$$

have been introduced.

Equation (6) has the same structure as the free Dirac equation; hence for the positive-energy eigenvalue one has

$$f_0^2(P) = \mathbf{p}^2 + \widetilde{m}^2(P) , \tag{9}$$

which implicitly yields, using eq. (7), the new dispersion relation for the renormalized energy $p_0 = \epsilon(\mathbf{p})$ of the nucleon in the nuclear medium:

$$p_0 = \frac{1-C(P)}{1-B(P)}\sqrt{\mathbf{p}^2+\widetilde{m}^2(P)} . \tag{10}$$

The corresponding positive-energy spinor reads (see refs. [5,6] for details)

$$\widetilde{\phi}_s(\mathbf{p}) \equiv \widetilde{u}_s(\mathbf{p},\widetilde{m}(\mathbf{p})) = \sqrt{Z_2(\mathbf{p})}\left(\frac{\widetilde{E}(\mathbf{p})+\widetilde{m}(\mathbf{p})}{2\widetilde{m}(\mathbf{p})}\right)^{1/2} \times \begin{bmatrix} \chi_s \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\widetilde{E}(\mathbf{p})+\widetilde{m}(\mathbf{p})}\chi_s \end{bmatrix} = \sqrt{Z_2(\mathbf{p})}u_s(\mathbf{p},\widetilde{m}(\mathbf{p})) , \tag{11}$$

where the function $\widetilde{m}(\mathbf{p})$ of the three-momentum $\mathbf{p}$ is obtained from the Dirac mass in eq. (8) by setting $p_0 = \epsilon(\mathbf{p})$

$$\widetilde{m}(\mathbf{p}) \equiv \widetilde{m}(\epsilon(\mathbf{p}),\mathbf{p}) \tag{12}$$

and

$$\widetilde{E}(\mathbf{p}) \equiv f_0(\epsilon(\mathbf{p}),\mathbf{p}) = \sqrt{\mathbf{p}^2+\widetilde{m}^2(\mathbf{p})} \tag{13}$$

represents the nucleon's Dirac energy. The field strength renormalization constant, $\sqrt{Z_2(\mathbf{p})}$, in eq. (11) is obtained from the renormalized nucleon propagator [6,11] and reads

$$Z_2(\mathbf{p}) = \mathrm{Res}\left.\frac{1}{[1-C(P)][f_0(P)-\widetilde{E}(P)]}\right|_{p_0=\epsilon(\mathbf{p})} = \left[1 - B - p_0\frac{\partial B}{\partial p_0} - m\frac{\widetilde{m}}{\widetilde{E}}\frac{\partial A}{\partial p_0} + \frac{\mathbf{p}^2}{\widetilde{E}}\frac{\partial C}{\partial p_0}\right]^{-1}_{p_0=\epsilon(\mathbf{p})} . \tag{14}$$

In the relativistic Hartree-Fock model the free spinors are used to compute the first approximation to the self-energy. This is then inserted in the Dirac equation to get new spinors, and so on. This self-consistent procedure has to be dealt with numerically.

The resulting momentum distribution is then obtained with the renormalized wave functions

$$\widetilde{\psi}_{\mathbf{k},s}(\mathbf{p}) = \int_V \mathrm{d}\mathbf{r}\, \widetilde{\psi}_{\mathbf{k},s}(\mathbf{r})e^{-i\mathbf{p}\cdot\mathbf{r}} = \sqrt{\frac{V\widetilde{m}(\mathbf{p})}{\widetilde{E}(\mathbf{p})}}\, \widetilde{u}_s(\mathbf{p},\widetilde{m}(\mathbf{p}))\delta_{\mathbf{k},\mathbf{p}} \tag{15}$$

and reads

$$\widetilde{n}(\mathbf{p}) = \sum_{\mathbf{k},s} \widetilde{\psi}^\dagger_{\mathbf{k},s}(\mathbf{p})\widetilde{\psi}_{\mathbf{k},s}(\mathbf{p})\theta(\widetilde{k}_{\rm F}-p) , \tag{16}$$

where $\widetilde{k}_{\rm F}$, $\widetilde{m}(\mathbf{p})$ and $\widetilde{E}(\mathbf{p})$ are the nucleon's renormalized Fermi momentum, mass and energy, respectively. From eqs. (16),(15),(11) the HF momentum distribution is then found to be

$$\widetilde{n}(\mathbf{p}) = 2VZ_2(\mathbf{p})\theta(\widetilde{k}_{\rm F}-p) , \tag{17}$$

which clearly reproduces the free result in eq. (3) for $Z_2(\mathbf{p}) = 1$ and $\widetilde{k}_{\mathrm{F}} = k_{\mathrm{F}}$.

Note that the HF wave function with the spinor (11) is *not* normalized to unity. Indeed

$$\int_V \mathrm{d}\mathbf{r} \widetilde{\psi}^\dagger_{\mathbf{k},s}(\mathbf{r}) \widetilde{\psi}_{\mathbf{k},s}(\mathbf{r}) = \frac{\widetilde{m}(\mathbf{p})}{\widetilde{E}} \widetilde{u}^\dagger_s(\mathbf{k}, \widetilde{m}(\mathbf{p})) \widetilde{u}_s(\mathbf{k}, \widetilde{m}(\mathbf{p})) = Z_2(\mathbf{k}) \ . \quad (18)$$

However, the total number of nucleons must be conserved. This implies that the unitarity condition

$$\int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \widetilde{n}(\mathbf{p}) = 2V \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} Z_2(\mathbf{p}) \theta(\widetilde{k}_{\mathrm{F}} - p) = Z \quad (19)$$

must be fulfilled. Equation (19) can be viewed as the procedure to fix the HF Fermi momentum $\widetilde{k}_{\mathrm{F}}$, which can in principle be different from the free one.

Here we consider the first-order correction to the momentum distribution arising from the HF series. We shall focus on mesonic correlations, induced by the exchange of π, ρ, ω and σ, associated with the following interaction Lagrangian [12]:

$$\mathcal{L}(x) = \bar{\psi}(x) \left\{ \frac{g_\pi}{2m} \gamma^5 \gamma^\mu \boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi}(x) + g_\rho \left[\gamma^\mu - \frac{a_\rho}{m} \sigma^{\mu\nu} \partial_\nu \right] \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu(x) + g_\omega \gamma^\mu \omega_\mu(x) + g_\sigma \sigma(x) \right\} \psi(x) \ . \quad (20)$$

Using this Lagrangian, we compute the self-energy in OBE approximation. For each meson $i = \pi, \rho, \omega, \sigma$, the corresponding self-energy functions $A_i(P), B_i(P), C_i(P)$ are given in the appendix. The total self-energy is obtained from eq. (5) with $A(P) = \sum_i A_i(P)$, $B(P) = \sum_i B_i(P)$ and $C(P) = \sum_i C_i(P)$. While the pion and rho self-energies correspond to purely exchange (Fock) terms, the sigma and omega also have a direct (Hartree) contribution due to their isoscalar nature.

The HF energy $\epsilon(\mathbf{p})$, the solution of eq. (10), can be computed analytically to first order in the squared meson-nucleon coupling constant g_i^2. For this purpose we note that the functions $A_i(P)$, $B_i(P)$ and $C_i(P)$ are of order $O(g_i^2)$. Hence, the following expansion of the Dirac mass in eq. (8) holds:

$$\widetilde{m}(P) = m \left[1 + A(P) + C(P) \right] + O(g_i^4) \ . \quad (21)$$

Inserting this into eq. (10) and expanding the right-hand side to first order in g_i^2, we get the equation

$$p_0 \simeq E_{\mathbf{p}} + \Delta E(P) \ , \quad (22)$$

where

$$\Delta E(P) = \frac{1}{E_{\mathbf{p}}} \left[m^2 A(P) + E^2_{\mathbf{p}} B(P) - \mathbf{p}^2 C(P) \right] \ . \quad (23)$$

Next we insert the value of p_0 given by eq. (22) inside the functions $A(P)$, $B(P)$, $C(P)$ and expand them around the on-shell value $p_0 = E_{\mathbf{p}}$, neglecting terms of second order in g_i^2. We get

$$A(P) \simeq A(E_{\mathbf{p}} + \Delta E, \mathbf{p}) \simeq A(E_{\mathbf{p}}, \mathbf{p}) \equiv A_0(\mathbf{p}) \quad (24)$$

and likewise $B(P) \simeq B_0(\mathbf{p})$, $C(P) \simeq C_0(\mathbf{p})$. Inserting these on-shell values into eq. (23), we finally obtain the HF energy to first order

$$p_0 \simeq E_{\mathbf{p}} + \frac{1}{E_{\mathbf{p}}} \left[m^2 A_0(\mathbf{p}) + E^2_{\mathbf{p}} B_0(\mathbf{p}) - \mathbf{p}^2 C_0(\mathbf{p}) \right] = \epsilon(\mathbf{p}) \ . \quad (25)$$

We proceed now by expanding as well the renormalized wave function, see eqs. (15),(11). For this purpose we expand the Dirac mass in eq. (21) around the on-shell energy

$$\widetilde{m}(\mathbf{p}) \simeq m \left[1 + A_0(\mathbf{p}) + C_0(\mathbf{p}) \right] \quad (26)$$

and likewise the Dirac energy $\widetilde{E}(\mathbf{p})$ defined in eq. (13)

$$\widetilde{E}(\mathbf{p}) = \frac{1-B}{1-C} \epsilon(\mathbf{p}) \simeq E_{\mathbf{p}} + \frac{m^2}{E_{\mathbf{p}}} \left[A_0(\mathbf{p}) + C_0(\mathbf{p}) \right] \ . \quad (27)$$

Moreover, for the field strength renormalization of eq. (14), we obtain

$$Z_2(\mathbf{p}) \simeq 1 + \alpha(\mathbf{p}) \quad (28)$$

with

$$\alpha(\mathbf{p}) \equiv B_0(\mathbf{p}) + \left[\frac{m^2}{E_{\mathbf{p}}} \frac{\partial A}{\partial p_0} + E_{\mathbf{p}} \frac{\partial B}{\partial p_0} - \frac{\mathbf{p}^2}{E_{\mathbf{p}}} \frac{\partial C}{\partial p_0} \right|_{p_0 = E_{\mathbf{p}}} \ . \quad (29)$$

After some algebra the following first-order expression for the HF wave function is obtained:

$$\widetilde{\psi}_{\mathbf{k},s}(\mathbf{p}) \simeq \sqrt{\frac{m}{E_{\mathbf{p}}}} \left[1 + m \frac{A_0(\mathbf{p}) + C_0(\mathbf{p})}{E_{\mathbf{p}}} \times \frac{E_{\mathbf{p}} \gamma_0 - m}{2E_{\mathbf{p}}} + \frac{1}{2} \alpha(\mathbf{p}) \right] u_s(\mathbf{p}, m) \delta_{\mathbf{k},\mathbf{p}} \ . \quad (30)$$

The above expansion transparently displays the effect of the self-energy on the free wave function. Indeed the second term in the square brackets of eq. (30) corresponds to a negative energy component with momentum $\mathbf{p}$. Thus, within the OBE potential approach the renormalized HF spinors in the nuclear medium are characterized by two new elements with respect to the bare $u_s(\mathbf{p}, m)$: the term $(E_{\mathbf{p}} \gamma_0 - m) u_s(\mathbf{p}, m)$, directly connected with the negative energy components in the wave function, and the term $\alpha(\mathbf{p})$, arising from the field strength renormalization $\sqrt{Z_2(\mathbf{p})}$. However, the negative energy component does not contribute to the momentum distribution in first order, where one simply gets

$$\widetilde{n}(\mathbf{p}) \simeq 2V \left[1 + \alpha(\mathbf{p}) \right] \theta(k_{\mathrm{F}} - p) \ . \quad (31)$$

The explicit expression for the first-order expansion of the function $\alpha(\mathbf{p}) \equiv \alpha(p) = \sum_i \alpha_i(p)$ is given in the appendix. Note that the Hartree self-energy of the ω and

σ does not contribute to α. The unitarity condition of eq. (19) becomes

$$2V \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} Z_2(\mathbf{p})\theta(\widetilde{k}_\mathrm{F} - p) = \frac{V\widetilde{k}_\mathrm{F}^3}{3\pi^2} + \frac{V}{\pi^2}\int_0^{\widetilde{k}_\mathrm{F}} p^2 \mathrm{d}p\alpha(p) = Z \ , \qquad (32)$$

which is certainly satisfied by $\widetilde{k}_\mathrm{F} = k_\mathrm{F}$, because the function α exactly satisfies

$$\int_0^{k_\mathrm{F}} \mathrm{d}p\, p^2 \alpha(p) = 0 \qquad (33)$$

(see appendix). We have numerically checked that there is no other value of $\widetilde{k}_\mathrm{F}$ for which the number of particles is Z. Therefore, *to first order in g_i^2 the Fermi momentum is not affected by the Hartree-Fock field.* This means that the present calculation respects not only Lorentz covariance, but also unitarity.

In fig. 1 we plot the first-order Dirac mass of eq. (26) (top panel) and the first-order momentum distribution of eq. (31) (bottom panel) as functions of p/k_F for $k_\mathrm{F} = 250$ MeV/c. The separate contributions of the various mesons are displayed. In the present calculation we empirically account for the short-range physics through the meson-nucleon form factors $F_i(\mathbf{k}) = \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 + \mathbf{k}^2}$ which cut off the nucleon-nucleon interaction in a spatial region of size $\sim 1/\Lambda_i$. Actually, for the sake of simplicity, we have approximated their effect by multiplying the self-energy associated with each meson by a constant factor (1 for the pion, 0.9 for the sigma, 0.5 for the omega and 0.4 for the rho): the form factors are indeed slowly varying functions of the meson momentum in the integration domain. The figure shows that the most sizable contribution to the Dirac mass arises from the σ-meson, which reduces the mass by about 30%, whereas the impact of the other mesons is at most 10% (in particular the pion induces a negligible increase of the mass): the total effect in the present model is a reduction of the mass by about a factor 0.6, in accord with the findings of refs. [7,10,13]. It is also remarkable that the p-dependence of the Dirac mass is almost negligible.

As far as the momentum distribution is concerned, it appears that σ, carrying an attractive interaction, induces a depletion of the baryonic density at low momenta and an enhacement of the latter in the vicinity of the Fermi surface, in contrast with the effect of the other mesons. It is interesting to note that the size of the Fock contribution in the momentum distribution decreases as the meson mass increases. This is in agreement with the fact that, at least for quasielastic inclusive electroweak responses modeled as we do here, the forces carried by the heavier mesons can be reasonably well approximated by four-fermion point interactions. In this case the HF approximation can be expressed as a linear combination of Hartree terms, which, as previously mentioned, do not affect the momentum distribution. Furthermore, and notably, the contributions arising from ρ, σ and ω cancel almost exactly. Thus, the net effect of the full interaction coincides with the one obtained

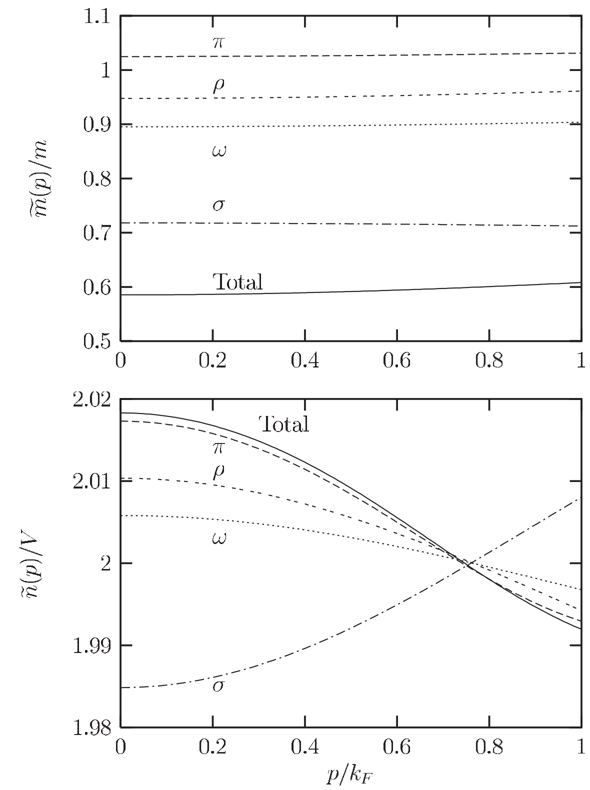

Fig. 1. The ratio $\widetilde{m}(\mathbf{p})/m$ of eq. (26) (upper panel) and the momentum distribution per unit volume of eq. (31) (lower panel) are plotted *versus* the nucleon momentum divided by the Fermi momentum ($k_\mathrm{F} = 250$ MeV/c). The results obtained by taking into account one single meson (pion: dashed lines; rho: double-dashed lines, omega: dotted lines; sigma: dot-dashed lines) are displayed together with the total result (solid lines). The mesonic parameters are [24]: $m_\pi = 139.6$ MeV/c, $m_\rho = 770$ MeV/c, $m_\omega = 782$ MeV/c, $m_\sigma = 550$ MeV/c, $g_\pi^2/4\pi^2 = 13.6$, $g_\rho^2/4\pi^2 = 0.84$, $a_\rho = 6.1$, $g_\omega^2/4\pi^2 = 20$, $g_\sigma^2/4\pi^2 = 7.78$, $\Lambda_\pi = 1720$ MeV/c, $\Lambda_\rho = 1310$ MeV/c, $\Lambda_\omega = 1500$ MeV/c, $\Lambda_\sigma = 2500$ MeV/c .

with the pion alone, and it amounts to an increase of the nucleon momentum density by about 1% for $p \simeq 0$ and to a decrease of it by almost the same amount for $p \simeq k_\mathrm{F}$, in such a way that the number of nucleons is conserved, according to the unitarity condition of eq. (19). It is worth noticing that the reduction of the momentum distribution near the Fermi surface due to relativistic HF correlations is of the same size as the one arising from short-range correlations of Jastrow type only, found in refs. [14,15] in the spectroscopic factors of quasihole valence states. Note, however, that this effect is very small compared with that expected from a more sophysticated non-relativistic modeling of short-range correlations [2], although one cannot make this statement with certainty in a relativistic context, since a relativistic version of Brueckner HF is even more challenging to carry out than relativistic HF and both constitute work for the future.

In fig. 2 the same observables displayed in fig. 1, the contributions of all the mesons being included, are shown for three values of the Fermi momentum: $k_\mathrm{F} = 200$ (solid

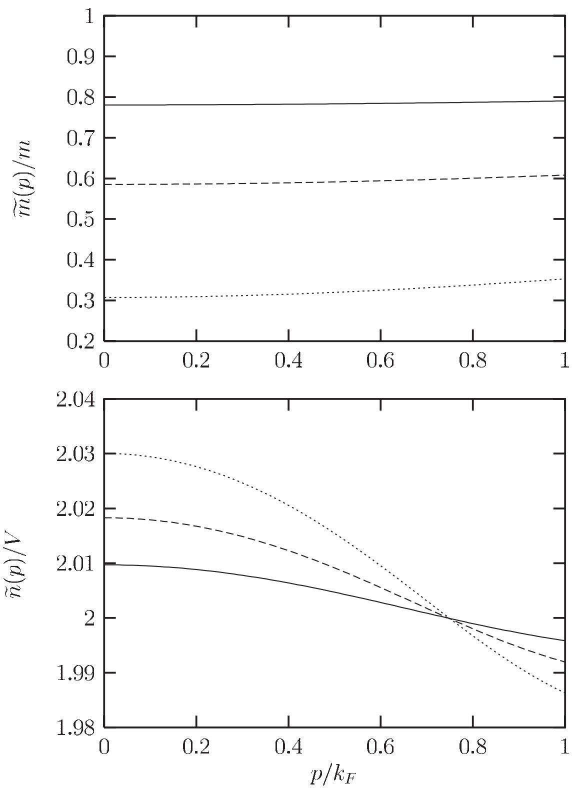

Fig. 2. The ratio $\widetilde{m}(\mathbf{p})/m$ of eq. (26) (upper panel) and the momentum distribution per unit volume of eq. (31) (lower panel) are plotted *versus* $p/k_{\rm F}$ for $k_{\rm F} = 200$ MeV/c (solid lines), $k_{\rm F} = 250$ MeV/c (dashed lines) and $k_{\rm F} = 300$ MeV/c (dotted lines).

lines), 250 (dashed lines) and 300 (dotted lines) MeV/c. It appears that the effect of the mesonic HF field on both the Dirac mass and the momentum distribution increases with the density and it is roughly proportional to $k_{\rm F}^3$. However, the origin of this dependence is different in the two cases, due to the different role played by the various mesons. Indeed the Dirac mass essentially stems from σ and ω, whereas the baryonic momentum density is significantly affected by all the four mesons.

A simple analysis of the $k_{\rm F}$-dependence of the OBE contributions to $\widetilde{m}(\mathbf{p})$ and $\widetilde{n}(\mathbf{p})$ can be performed through an expansion in the small parameter $\eta_{\rm F} = k_{\rm F}/m$, whose typical value is $\sim 1/4$. Such an expansion has been successfully applied to the study of inclusive and exclusive electron scattering both for free and correlated nuclear systems [16–20]. Remarkably, the $\eta_{\rm F}$ expansion has been shown to be very useful for exploring the role of chiral pion dynamics in nuclear matter [21,22].

When performing this expansion, one should pay attention to the fact that the pion is much lighter than the other mesons: this induces a different $k_{\rm F}$-dependence for the pionic contributions, since $k_{\rm F}/m_\pi$ cannot be treated as a small parameter. In fact it is easy to show that the heavy-mesons' contributions to $\widetilde{m}(\mathbf{p})$ go as $k_{\rm F}^3$ (the pion contribution is negligible). On the other hand, in $\widetilde{n}$ the pionic effect grows as $k_{\rm F}^3$, while the heavy mesons contribute as $k_{\rm F}^5$. We recall that σ, ω and ρ almost cancel in the momentum distribution (see fig. 1).

It is of importance to notice that the physics of real nuclei roughly corresponds to the range $200 \leq k_{\rm F} \leq 250$ MeV/c: here our prediction for the Dirac mass, $\widetilde{m}$, is close to the one for the effective mass [8,23]. It is only for larger $k_{\rm F}$ that the two quantities start to differ substantially.

Finally, it is also interesting to note that the momentum distribution $\widetilde{n}(p)$ coincides with the free one, $n(p) = 2$, for a value of $p/k_{\rm F}$, which is independent of both the specific meson and the value of $k_{\rm F}$ (see figs. 1 and 2). This finding can again be interpreted on the basis of the above-mentioned expansion, which shows that the function α vanishes for $p \simeq \sqrt{3/5}k_{\rm F}$.

Before drawing our conclusions, we would like to address the issue of the relevance of our findings on physical observables. In this regard we have shown in refs. [5,6] that the effect on the electromagnetic response functions including pionic correlations due to the modification of the momentum distribution is negligible (see in particluar fig. 13 in [6]). One could not say this *a priori* and so an important conclusion of that work plus the deeper understanding presented in this paper is that at the level where relativity is dealt with in a consistent way such correlations appear under typical circumstances to be perturbatively small.

In summary, in this paper we have presented a relativistic analysis of the single-particle properties of nuclear matter in HF approximation within a meson exchange model. In particular, we have focused on the role played by the pion, rho, omega and sigma on the Dirac mass of the nucleon and on the momentum distribution. Whereas the momentum distribution is not affected by the HF field in a non-relativistic framework, in the relativistic case it is slightly modified due to the renormalization of the spinors. In this work we have quantified this effect to first order in the coupling constant where the HF equations can be solved analytically. Using this solution we have demonstrated that the field strength renormalization function exactly satisfies unitarity at this order.

Moreover, whereas for the Dirac mass, as is well known, the effect of HF mesonic correlations amounts to about 30-40% and mainly arises from the σ- and ω-mesons, we have shown that in the momentum distribution a cancellation among the heavier mesons occurs and the total result basically coincides with the pionic contribution, which amounts to a 1–3% effect, depending upon the density.

This work was partially supported by funds provided by DGICYT (Spain) under Contract Nos. PB/98-1111, PB/98-0676 and PB/98-1367 and the Junta de Andalucía (Spain) and by the INFN-CICYT exchange, as well as by the U.S. Department of Energy under cooperative research agreement No. DE-FC02-94ER40818. M.B.B. acknowledges MEC (Spain) for a sabbatical stay at the University of Sevilla (ref. SAB2001-0025).

Appendix A.

The functions A, B, C of eq. (5) can be expressed in terms of the integrals

$$I(P, m_i) \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^3}\theta(k_{\mathrm{F}} - k) \times \frac{1}{2E_{\mathbf{k}}}\frac{1}{(P-K)^2 - m_i^2} , \tag{A.1}$$

$$L^\mu(P, m_i) \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^3}\theta(k_{\mathrm{F}} - k) \times \frac{1}{2E_{\mathbf{k}}}\frac{K^\mu}{(P-K)^2 - m_i^2} , \tag{A.2}$$

$$L^{\mu\nu}(P, m_i) \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^3}\theta(k_{\mathrm{F}} - k) \times \frac{1}{2E_{\mathbf{k}}}\frac{K^\mu K^\nu}{(P-K)^2 - m_i^2} , \tag{A.3}$$

through the following relations[1]:

$$A_\pi(P) = \frac{3g_\pi^2}{2}\left[I(P, m_\pi) - \frac{P_\mu L^\mu(P, m_\pi)}{m^2} + \frac{P^2 - m^2}{2m^2}I(P, m_\pi)\right] , \tag{A.4}$$

$$B_\pi(P) = \frac{3g_\pi^2}{2}\left[I(P, m_\pi) - \frac{P_\mu L^\mu(P, m_\pi)}{m^2} + \frac{P^2 - m^2}{2m^2}\frac{L_0(P, m_\pi)}{p_0}\right] , \tag{A.5}$$

$$C_\pi(P) = \frac{3g_\pi^2}{2}\left[I(P, m_\pi) - \frac{P_\mu L^\mu(P, m_\pi)}{m^2} + \frac{P^2 - m^2}{2m^2}\frac{L_3(P, m_\pi)}{p}\right] \tag{A.6}$$

for the pion,

$$A_\rho(P) = 6g_\rho^2\left[(2 + 3a_\rho + 3a_\rho^2)I(P, m_\rho) - 3a_\rho(1 + a_\rho)\frac{P_\mu L^\mu(P, m_\rho)}{m^2} + \frac{3a_\rho^2}{2m^2}(P^2 - m^2)I(P, m_\rho)\right] , \tag{A.7}$$

$$B_\rho(P) = 6g_\rho^2\Bigg\{(3a_\rho + 2a_\rho^2)I(P, m_\rho) - \frac{a_\rho^2}{m^2}P_\mu\left[2L^\mu(P, m_\rho) - \frac{L^{0\mu}(P, m_\rho)}{p_0}\right] - \left[1 + 3a_\rho + a_\rho^2 - \frac{a_\rho^2}{2m^2}(P^2 - m^2)\right]\frac{L_0(P, m_\rho)}{p_0}\Bigg\}, \tag{A.8}$$

$$C_\rho(P) = 6g_\rho^2\Bigg\{(3a_\rho + 2a_\rho^2)I(P, m_\rho) - \frac{a_\rho^2}{m^2}P_\mu\left[2L^\mu(P, m_\rho) - \frac{L^{3\mu}(P, m_\rho)}{p}\right] - \left[1 + 3a_\rho + a_\rho^2 - \frac{a_\rho^2}{2m^2}(P^2 - m^2)\right]\frac{L_3(P, m_\rho)}{p}\Bigg\} \tag{A.9}$$

for rho,

$$A_\omega(P) = 4g_\omega^2 I(P, m_\omega) , \tag{A.10}$$

$$B_\omega(P) = 2g_\omega^2\left[\frac{k_{\mathrm{F}}^3}{3p_0\pi^2 m_\omega^2} - \frac{L_0(P, m_\omega)}{p_0}\right] , \tag{A.11}$$

$$C_\omega(P) = -2g_\omega^2\frac{L_3(P, m_\omega)}{p} \tag{A.12}$$

for omega and

$$A_\sigma(P) = -g_\sigma^2\left[I(P, m_\sigma) + \frac{1}{\pi^2 m_\sigma^2} \times \left(k_{\mathrm{F}}E_{\mathrm{F}} - m^2\ln\frac{k_{\mathrm{F}} + E_{\mathrm{F}}}{m}\right)\right] , \tag{A.13}$$

$$B_\sigma(P) = -g_\sigma^2\frac{L_0(P, m_\sigma)}{p_0} , \tag{A.14}$$

$$C_\sigma(P) = -g_\sigma^2\frac{L_3(P, m_\sigma)}{p} \tag{A.15}$$

for sigma.

The corresponding expression for the functions α_i (see eq. (29)), with $\alpha = \sum_{i=\pi,\rho,\omega,\sigma}\alpha_i$, is

$$\alpha_i(p) = \frac{m_i^2 g_i^2}{4\pi^2 E_{\mathbf{p}}}\int_0^{k_{\mathrm{F}}}\mathrm{d}k\frac{k^2}{E_{\mathbf{k}}}\frac{E_{\mathbf{k}} - E_{\mathbf{p}}}{\gamma_i^2(p,k) - 4p^2k^2}f_i(p,k) , \tag{A.16}$$

where

$$\gamma_i(p,k) \equiv (E_{\mathbf{p}} - E_{\mathbf{k}})^2 - p^2 - k^2 - m_i^2 = 2m^2 - m_i^2 - 2E_{\mathbf{p}}E_{\mathbf{k}} \tag{A.17}$$

and the functions $f_i(p,k)$ are defined as

$$f_\pi(p,k) = 3 , \tag{A.18}$$

[1] **L** is parallel to **p** since, choosing **p** along the z-axis, the azimuthal integration in eq. (A.2) yields $L_1 = L_2 = 0$.

$$f_\rho(p,k) = 3\left[2(1+6a_\rho+4a_\rho^2) + a_\rho^2\frac{m_\rho^2}{m^2} + 4\frac{m^2}{m_\rho^2} + \left(2+6a_\rho+a_\rho^2\frac{m_\rho^2}{m^2}\right)\frac{\gamma_\rho^2(p,k)-4p^2k^2}{4kpm_\rho^2} \times \ln\frac{\gamma_\rho(p,k)+2kp}{\gamma_\rho(p,k)-2kp}\right], \quad (A.19)$$

$$f_\omega(p,k) = 2\left[1+2\frac{m^2}{m_\omega^2} + \frac{\gamma_\omega^2(p,k)-4p^2k^2}{2kpm_\omega^2} \times \ln\frac{\gamma_\omega(p,k)+2kp}{\gamma_\omega(p,k)-2kp}\right], \quad (A.20)$$

$$f_\sigma(p,k) = 1-4\frac{m^2}{m_\sigma^2} + \frac{\gamma_\sigma^2(p,k)-4p^2k^2}{4kpm_\sigma^2} \times \ln\frac{\gamma_\sigma(p,k)+2kp}{\gamma_\sigma(p,k)-2kp}. \quad (A.21)$$

The meson-nucleon form factors have been neglected for simplicity. Their impact on the results is discussed in the text. Using the above expressions, the unitarity condition of eq. (33) follows, since the functions $f_i(p,k)$ are all symmetrical under the exchange of p and k, hence

$$\int_0^{k_\mathrm{F}} \mathrm{d}p p^2\alpha_i(p) = \frac{m_i^2 g_i^2}{4\pi^2}\int_0^{k_\mathrm{F}}\mathrm{d}p\int_0^{k_\mathrm{F}}\mathrm{d}k\frac{p^2k^2}{E_\mathbf{p}E_\mathbf{k}} \times\frac{E_\mathbf{k}-E_\mathbf{p}}{\gamma_i^2(p,k)-4p^2k^2}f_i(p,k)=0. \quad (A.22)$$

References

1. A.L. Fetter, J.D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).
2. H. Müther, A. Polls, Prog. Part. Nucl. Phys. **45**, 243 (2000).
3. S. Fantoni, V. Pandharipande, Nucl. Phys. A **427** 1984 473.
4. W. Dickhoff, H. Müther, Rep. Prog. Phys. **55**, 1947 (1992).
5. J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, A. Molinari, Nucl. Phys. A **697**, 388 (2002).
6. J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, A. Molinari, Phys. Rep. **368**, 317 (2002).
7. B.D. Serot, J.D. Walecka, Adv. Nucl. Phys. **16**, 1 (1988).
8. L.S. Celenza, C.M. Shakin, *Relativistic Nuclear Physics* (World Scientific, Singapore, 1986).
9. J.D. Bjorken, S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1965).
10. M.R. Anastasio, L.S. Celenza, C.M. Shakin, Phys. Rev. **23**, 569 (1981).
11. M.E. Peskin, D.V. Schroeder, *An Introduction to Quantum Field Theory* (Perseus, 1995).
12. R. Machleidt, K. Holinde, Ch. Elster, Phys. Rep. **149**, No. 1 (1987) 1.
13. E. Schiller, H. Müther, Eur. Phys. J. A **11**, 15 (2001).
14. A. Fabrocini, G. Co', Phys. Rev. C **63**, 044302 (2001).
15. M. Mazziotta, J.E. Amaro, F. Arias de Saavedra, Phys. Rev. C **65**, 034602 (2002).
16. J.E. Amaro, J.A. Caballero, T.W. Donnelly, A.M. Lallena, E. Moya de Guerra, J.M. Udias, Nucl. Phys. A **602**, 263 (1996).
17. S. Jeschonnek, T.W. Donnelly, Phys. Rev. C **57**, 2438 (1998).
18. J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, A. Molinari, Nucl. Phys. A **643**, 349 (1998).
19. J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, A. Molinari, Nucl. Phys. A **657**, 161 (1999).
20. L. Álvarez-Ruso, M.B. Barbaro, T.W. Donnelly, A. Molinari, Phys. Lett. B **497**, 214 (2001).
21. N. Kaiser, S. Fritsch, W. Weise, Nucl. Phys. A **700**, 343 (2002).
22. N. Kaiser, S. Fritsch, W. Weise, Nucl. Phys. A **697**, 255 (2002).
23. M.B. Barbaro, A. De Pace, T.W. Donnelly, A. Molinari, Nucl. Phys. A **596**, 553 (1996).
24. R. Machleidt, Phys. Rev. C **63**, 024001 (2001).